3/2/18 "I make the most of all that comes and the least of all that goes."-Sara Teasdale

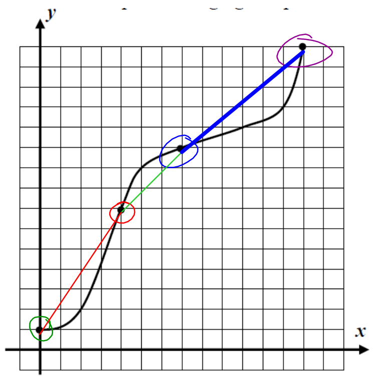
HW: "Average Rate of Change" homework section
Test 2 on Friday 3/9

AIM: How do we find Average Rates of Change?

Warm Up:

Determine the equation of the parabola whose focus is (0,8) and whose directrix is the horizontal line $y = 2$?

***Exercise* #1:** The function $f(x)$ is shown graphed to the right.



(a) Evaluate each of the following based on the graph:

(i) $f(0)$ $\boxed{1}$ (ii) $f(4)$ $\boxed{7}$ (iii) $f(7)$ $\boxed{10}$ (iv) $f(13)$ $\boxed{15}$

(b) Find the change in the function, Δf, over each of the following domain intervals. Find this both by subtraction and show this on the graph.

(i) $0 \le x \le 4$

$y=1$ $y=7$

$\Delta f = 6$

(ii) $4 \le x \le 7$

$y=7$ $y=10$

$\boxed{\Delta f = 3}$

(iii) $7 \le x \le 13$

$y=10$ $y=15$

$\boxed{\Delta f = 5}$

(c) Why can't you simply compare the changes in f from part (b) to determine over which interval the function changing the fastest?

Intervals are not the same

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate.

$\text{Slope} = \frac{\Delta f}{\Delta x}$

(i) $0 \le x \le 4$

$\frac{\Delta f}{\Delta x} = \frac{6}{4} = 1.5$

Average rate of change $\boxed{1.5}$ ← Greatest

(ii) $4 \le x \le 7$

$\frac{\Delta f}{\Delta x} = \frac{3}{3}$

AROC = 1

(iii) $7 \le x \le 13$

$\frac{\Delta f}{\Delta x} = \frac{5}{6}$

AROC = .833

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

Average rate of change is the slope of the line Connecting points.

AVERAGE RATE OF CHANGE

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b)-f(a)}{b-a} = \frac{y-y_1}{x-x_2}$$

***Exercise* #2:** Consider the two functions $f(x) = 5x + 7$ and $g(x) = 2x^2 + 1$.

Average rate of change = slope

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

$\frac{\Delta y}{\Delta x}$

$f(x) = \boxed{5}$ (line)

$g(-2) = 9$

$g(3) = 19$

$\frac{9-19}{-2-3} = \frac{-10}{-5} = \boxed{2}$

(ii) $1 \le x \le 5$

$f(x) = 5 \leftarrow$ AROC (line)

$g(1) = 3$ (x, y)

$g(5) = 51$

$\frac{51-3}{5-1} = \frac{48}{4} = \boxed{12}$ AROC

(b) The average rate of change for f was the same for both (i) and (ii) but was not the same for g. Why is that?

"f" is a line which has a constant rate of change

"g" is a parabola that changes at multiple rates

***Exercise* #3:** The table below represents a linear function. Fill in the missing entries.

rate of change is a constant

x	1	5	11	x	45
y	-5	1	y	22	z

$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-5-1}{1-5} = \frac{-6}{-4} = \frac{3}{2} \leftarrow$ use the slope and one of the given points

$\frac{3}{2} = \frac{1-y}{5-11}$

$(-6)\frac{3}{2} = \frac{1-y}{-6} \cdot -6$

$\frac{-18}{2} = 1-y$

$-9 = 1-y$

$-1 \quad -1$

$-10 = -y$

$\boxed{y=10}$

$\frac{3}{2} = \frac{22-1}{x-5}$

$\frac{3}{2} = \frac{21}{x-5}$

$\frac{42}{3} = \frac{3(x-5)}{3}$

$14 = x-5$

$\boxed{19 = x}$

$\frac{3}{2} = \frac{z-1}{45-5}$

$(40)\frac{3}{2} = \frac{z-1}{40}(40)$

$\frac{120}{2} = z-1$

$60 = z-1$

$\boxed{z=61}$